Parachute Flight Dynamics and Trajectory Simulation

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based on lectures presented at the "Heinrich Parachute Systems Short Course", University of St. Louis, 2002

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- Knacke, T.W., "Parachute Recovery Systems Design Manual", NWC TP 6575, Para Publishing, Santa Barbara, CA, 1992.
Remark: Figs. 5-46 and 5-50 have been used in the following lecture.

- Cockrell, D.J., "The Aerodynamics of Parachutes", AGARDograph No. 295, 1987.

- Wolf, D.F., "The Dynamic Stability of a Non-Rigid-Parachute and Payload System", J. Aircraft, Vol. 8, No. 8, August 1971, pp 603-609.

- Doherr, K.-F. ; Schilling, H., "Nine-Degree-of-Freedom Simulation of Rotating Parachute Systems", J. Aircraft, Vol. 29, No. 5, Sept.-Oct. 1992.

- Doherr, K.-F., "Extended Parachute Opening Shock Estimation Method", AIAA 2003-2173, 17th Aerodynamic Decelerator Systems Technology Conference and Seminar, 19-22 May 2003, Monterey, California.

Some Literature

Questions:

- Trajectory
Where is the parachute-payload system at what time?

- Force history
What are the peak forces?

- Dynamic Stability
Does the system oscillate?

My Strategy:

Offering you a Bundle of Illusions by:

- **Setting up Mathematical Models**
- **Presenting some Closed-form Solutions**
- **Applying Computer Codes**

Why Illusions?

1. Parachutes are Stochastic Systems
with large scatter of their performance characteristics

2. The atmosphere is of stochastic character
(gusts, wind-shear, up- and down-winds in the order of the parachute velocity of descent)

3. There are almost never enough experiments
to validate the mathematical models
(due to lack of time, money, test equipment, staff etc.)

4. Parachutes are literaly too cheap
to attract big research money and public interest

So, what will you get today?

Some tools to study the effect of selected parachute parameters

Example:

Cylindrical Payload (L) released from climbing aircraft at

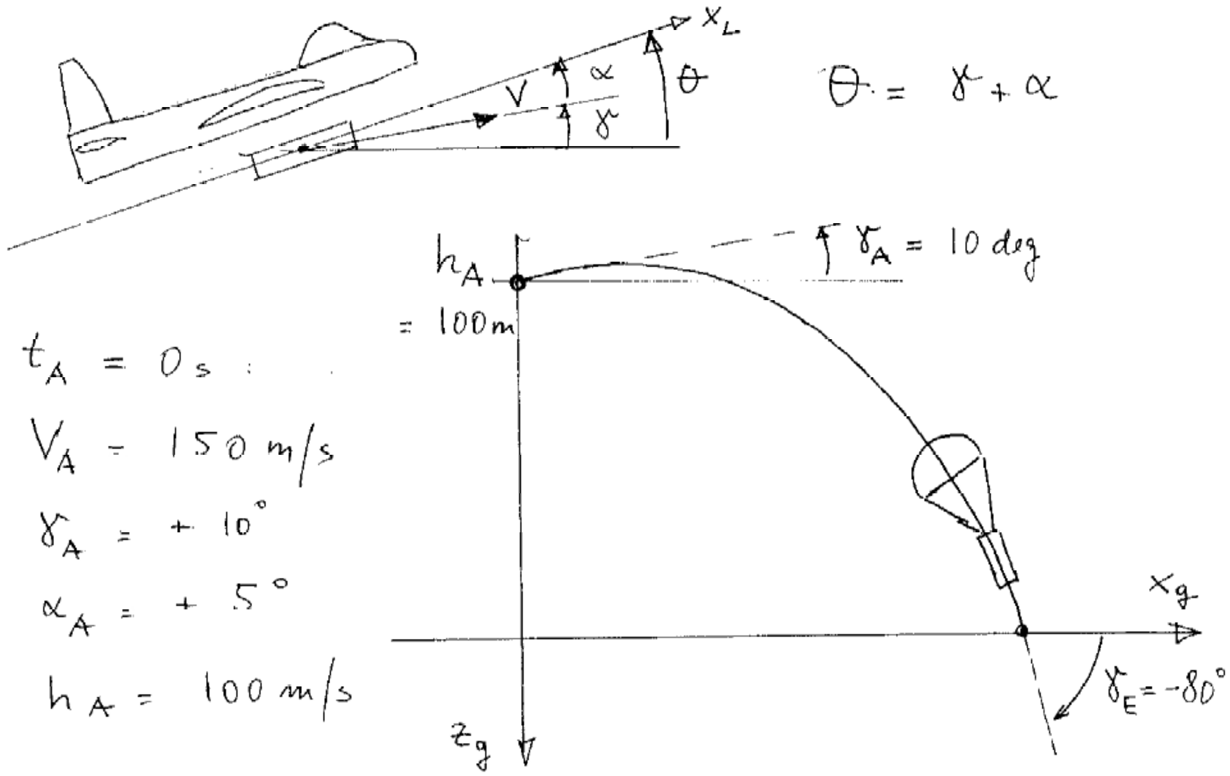
h_A = 100 m, V_A = 150 m/s
γ_A = + 10°, α_A = + 5°, Θ_A = +15°

shall be decelerated by a parachute
and land at steady state velocity V_e = 27.3 m/s

Find / calculate:

1. suitable parachute
2. time it takes to achieve steady state velocity
3. snatch force
4. opening shock (max inflation force)
5. angle of attack oscillations, horizontal and vertical
6. trajectory

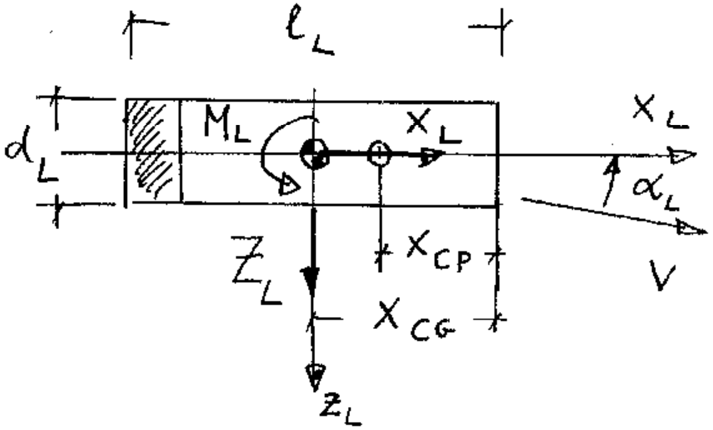
Study Case



Study Case : Example

Cylindrical payload:

$m_L = 40$ kg
$d_L = 0.2$ m
$\ell_L = 0.8$ m
$I_{xx} = 0.2$ kg m^2
$I_{yy} = 2.1$ kg m^2



$$C_{XL} = \frac{X_L}{\varrho/2\ V^2 S_L} = -1.0$$

$$C_{ZL\alpha} = \frac{Z_{L\alpha}}{\varrho/2\ V^2 S_L} = -2.78$$

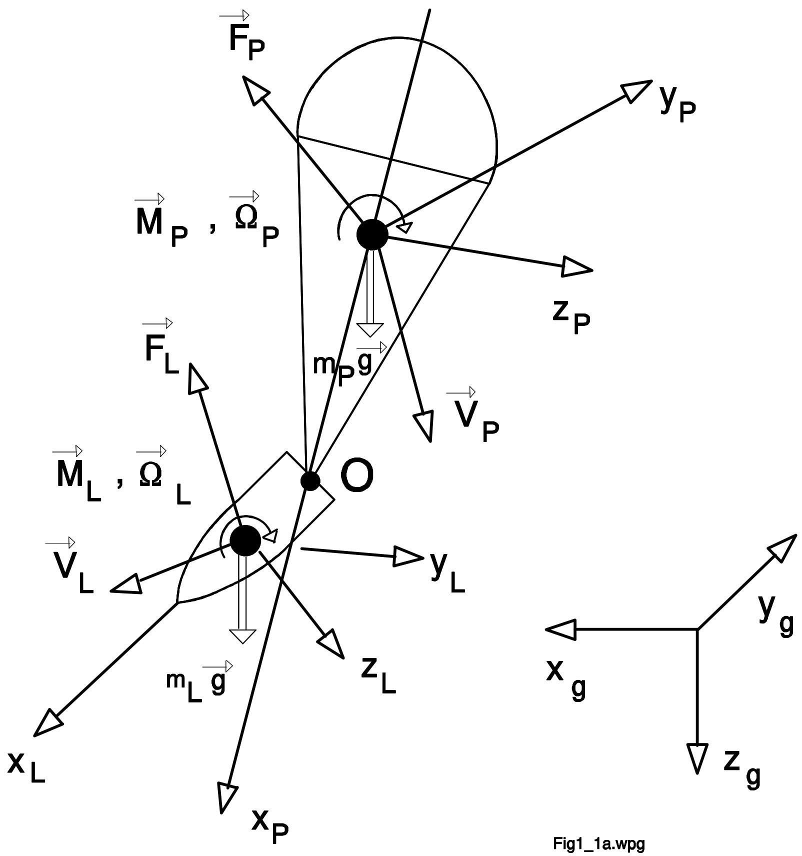
$$C_{mL\alpha} = \frac{M_{L\alpha}}{\varrho/2\ V^2 S_L d_L} = +1.11$$

$$C_{mLq} = \frac{M_{L\hat{q}}}{\varrho/2\ V^2 S_L d_L} = -2.0$$

$$\frac{x_{CP}}{\ell_L} = 0.4$$

$$\frac{x_{CG}}{\ell_L} = 0.5$$

Study Case



- Rigid body mathematical models get already very complex with 9 DoF =

6 DoF of payload +
3 DoF of parachute rotating relative to payload

- We will consider simplified systems with 2 DoF and 3 DoF

- Most important design parameter:
Drag area C_DS:

$$D = C_D S \frac{\rho}{2} V^2$$

Parachute - Payload System

Trajectory analysis		Point Mass	Planar Rigid Body	6DOF Rigid Body	9DOF 2 Rigid Bodies
Degrees of freedom DOF		2	3	6	9
Major Variables		x, z	x, z, Θ	x, y, z, ψ, Θ, Φ	x, y, z, ψ, Θ, Φ, ψ_P, Θ_P, Φ_P
Decelerator Input	Mass	▪	▪	▪	■
	Inertias		▪	▪	■
	C_DS (drag area)	■	■	■	■
	C_N (normal)		▪	■	■
	C_l (roll)			■	■
	X_{CP} (center of pressure)		▪	▪	■
	α_{ij} (apparent masses)		▪	▪	■
Coupling Conditions					■

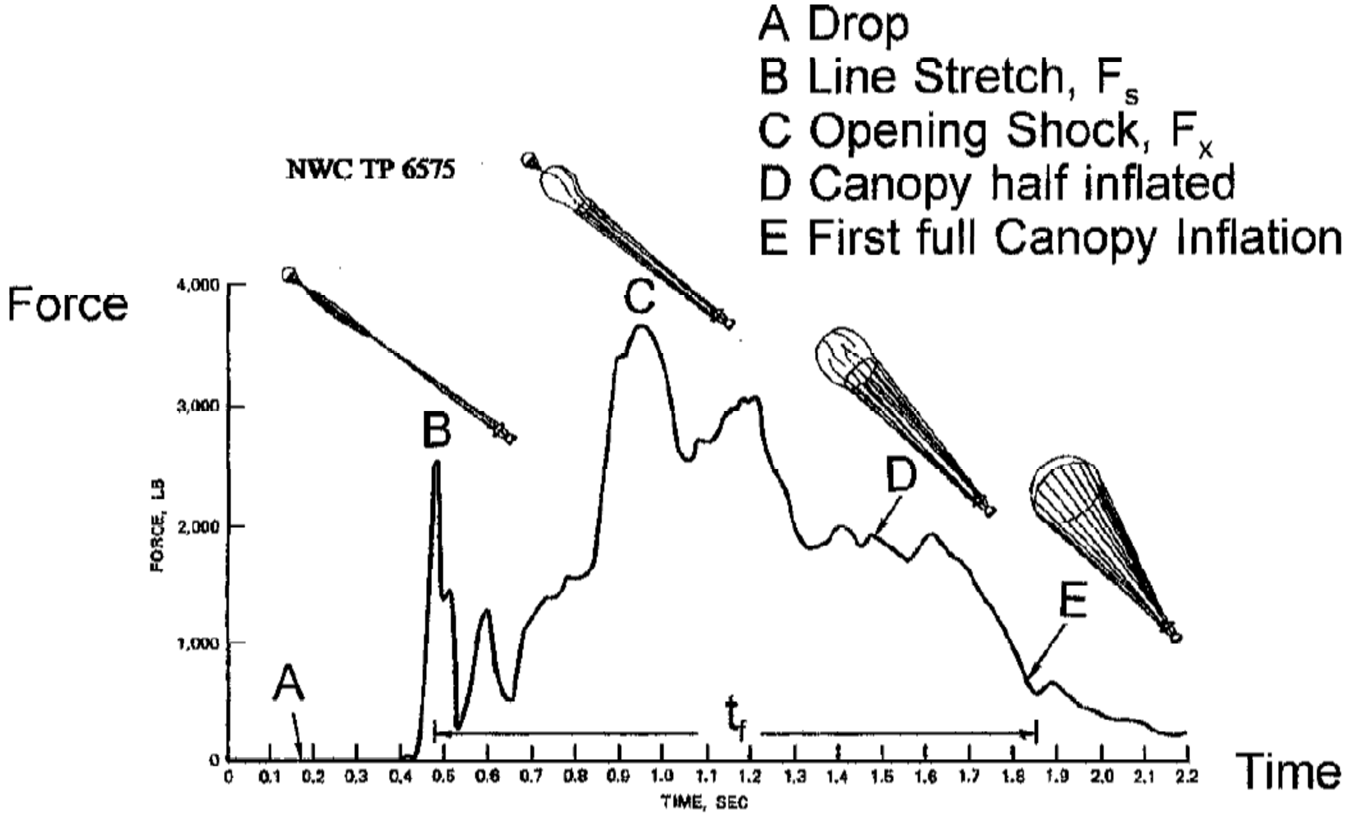
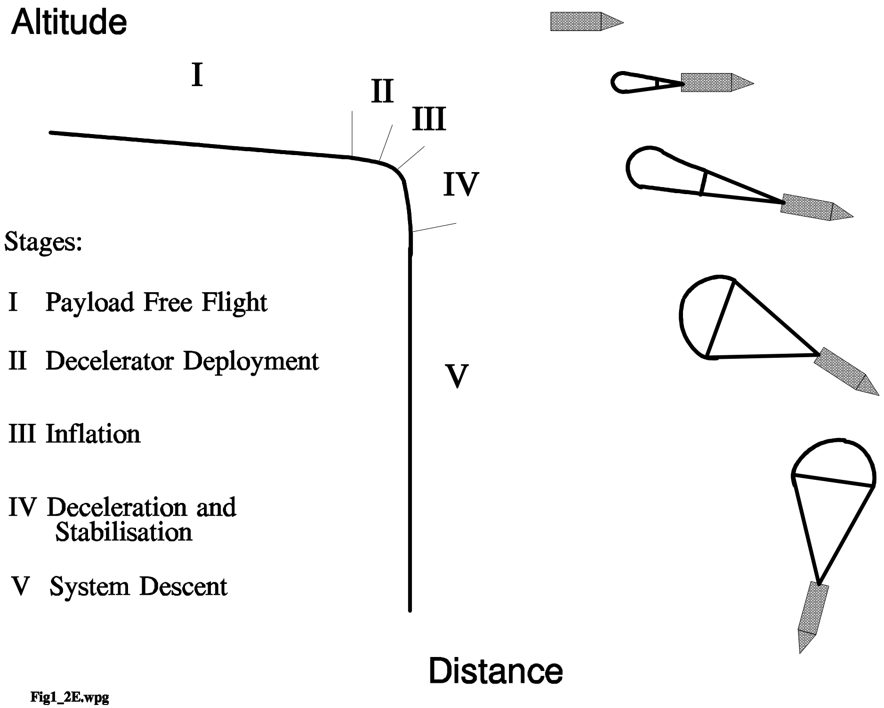


FIGURE 5-46. Opening Process and Opening Force Versus Time for a Guide Surface Personnel Parachute Tested at the El Centro Whirl Tower at 250 Knots With a 200-Pound Torso Dummy.

Parachute Opening Force / Inflation Force



I Payload free flight
C_DS = const

II Parachute deployment to fully-stretched rigging; snatch force F_S

III Inflation:
$C_DS = f(t)$
opening shock F_x

IV Deceleration and Stabilization
C_DS -> const
oscillation?

V System Descent
C_DS = const

System Flight Stages

Stage	Analytical Solutions	Computer Programs
I Free Flight	Point mass 2 DoF $C_D S$ = const - horizontal / vertical flight	2DOFT05
II Deployment	Two point masses, each 1 DoF --> Snatch force F_s	SNATCH
III Inflation	$C_D S = f(t)$ Pflanz' approximation --> Opening shock $F_X = F(t)_{max}$	Fx05 2DOFT05
IV Deceler and Stabilization	$C_D S$ = const $\alpha = \alpha(t)$; 3 DoF	OSCILALH OSCILALV
V Descent	$C_D S$ = const	2DOFT05

Plan of Attack

I Payload Free Flight

V System Steady Descent

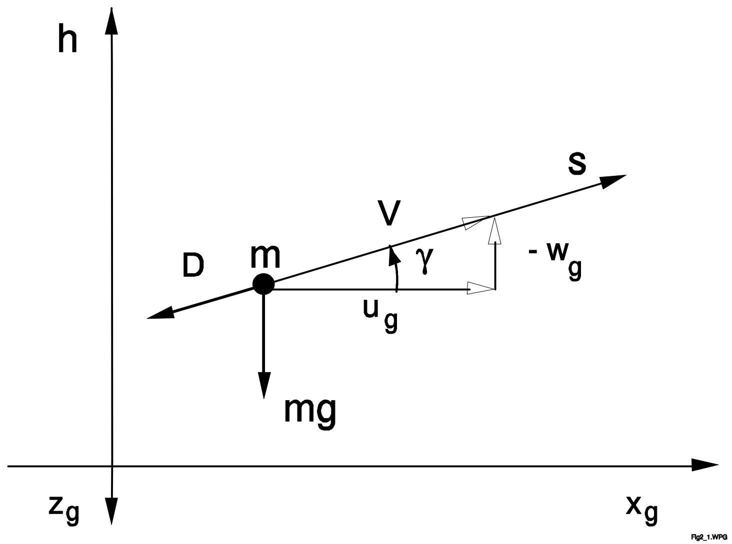
with constant drag area C_DS

Equations of motion in earth fixed coordinates (2DOFT05)

Non-linear differential equations:

$$m\,\dot{u}_g = -\,D\cos\gamma$$

$$m\,\dot{w}_g = D\sin\gamma + m\,g$$



$$\dot{x}_g = u_g$$

$$\dot{z}_g = w_g$$

Initial conditions:

$$t = t_A: \quad x_g = x_A; \quad z_g = z_A; \quad u_g = u_A; \quad w_g = w_A$$

Non-linear algebraic relations:

$$\sin\gamma = -\, w_g / V$$ flight path angle

$$\cos\gamma = u_g / V$$

$$D = C_D S\ \rho/2\ V^2$$ drag

$$C_D S(t) = (C_D S)_L + (C_D S)_P$$ drag area

$$\rho = \rho(h)$$ density

atmosphere wind, thermal upwind, gusts

Simplified cases:

C_DS = const., ρ = const, $V_W = 0$
(no parachute; or parachute undeployed; or parachute fully open; small changes in altitude; no wind):

horizontal flight:

$$\gamma = 0^\circ \,;\; u_g = V \,;\; w_g = 0$$

$$m\dot{V} = -\frac{\rho}{2} C_D S \; V^2$$

$$m\dot{w}_g = mg$$

vertical flight:

$$\gamma = -90^\circ \,;\; u_g = 0 \,;\; w_g = V$$

$$m\dot{u}_g = 0$$

$$m\dot{V} = mg - \frac{\rho}{2} C_D S \; V^2$$

Vertical flight:

System decelerates (or accelerates) until drag becomes equal weight:

$V \dashrightarrow V_e$

$$0 = -D + mg$$

equilibrium for $t \to \infty$

$$mg = \frac{\rho}{2} V_e^2 C_D S$$

$$V_e = \sqrt{\frac{2mg}{\rho C_D S}}$$

steady state velocity = velocity of descent

System parameters (m, ϱ, C_D, S, g) can be replaced by one parameter V_e only!

horizontal flight

$$\frac{1}{g}\frac{dV}{dt} = -\frac{V^2}{V_e^2}$$

$$\hat{V} = \frac{\hat{V}_A}{1 + \hat{V}_A \Delta \hat{t}}$$

$$\Delta \hat{x} = \ln (1 + \hat{V}_A \Delta \hat{t})$$

vertical flight

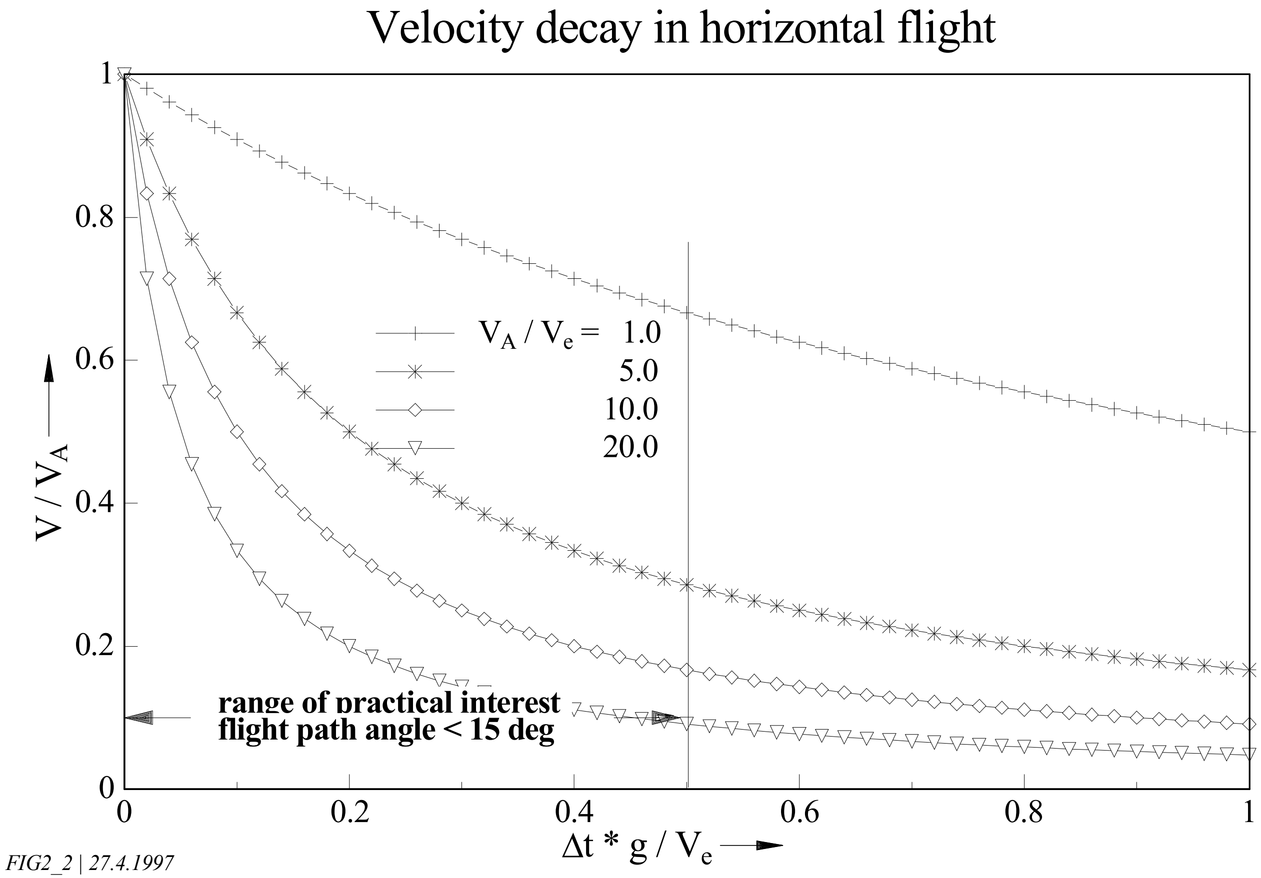
$$\frac{1}{g}\frac{dV}{dt} = 1 - \frac{V^2}{V_e^2}$$

$$\hat{V} = \frac{1 + a * e^{-2 \Delta \hat{t}}}{1 - a * e^{-2 \Delta \hat{t}}}$$

$$\Delta \hat{z} = \Delta \hat{t} + \ln \frac{1 - a * e^{-2 \Delta \hat{t}}}{1 - a}$$

where: $\hat{V} = V / V_e$; $\hat{t} = t * g / V_e$; $\hat{x} = x_g * g / V_e^2$; $\hat{z} = z_g * g / V_e^2$

$$a = (\hat{V}_A - 1) / (\hat{V}_A + 1)$$


Velocity decay in horizontal flight
V / V_A
Δt * g / V_e
V_A / V_e = 1.0
5.0
10.0
20.0
range of practical interest
flight path angle < 15 deg
0
0.2
0.4
0.6
0.8
1
FIG2_2 | 27.4.1997

Distance travelled in horizontal flight

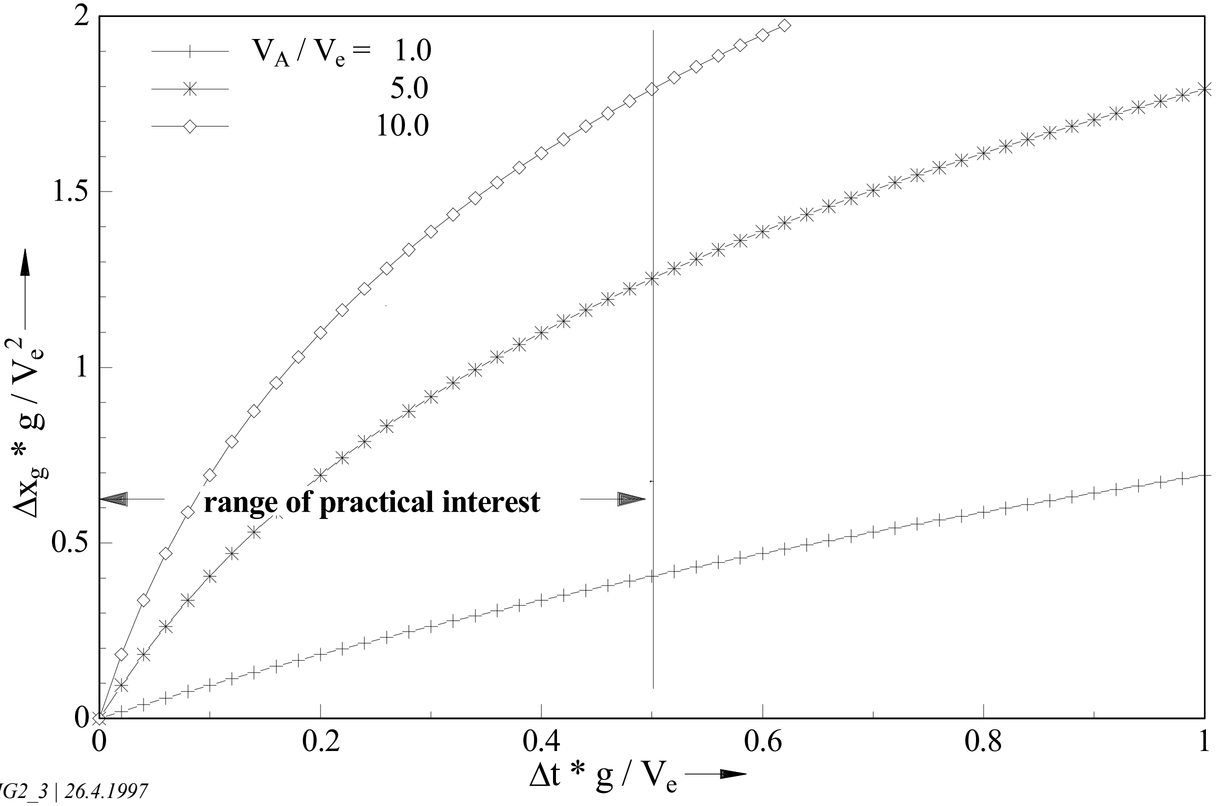


FIG2_3 | 26.4.1997

Velocity decay in vertical flight

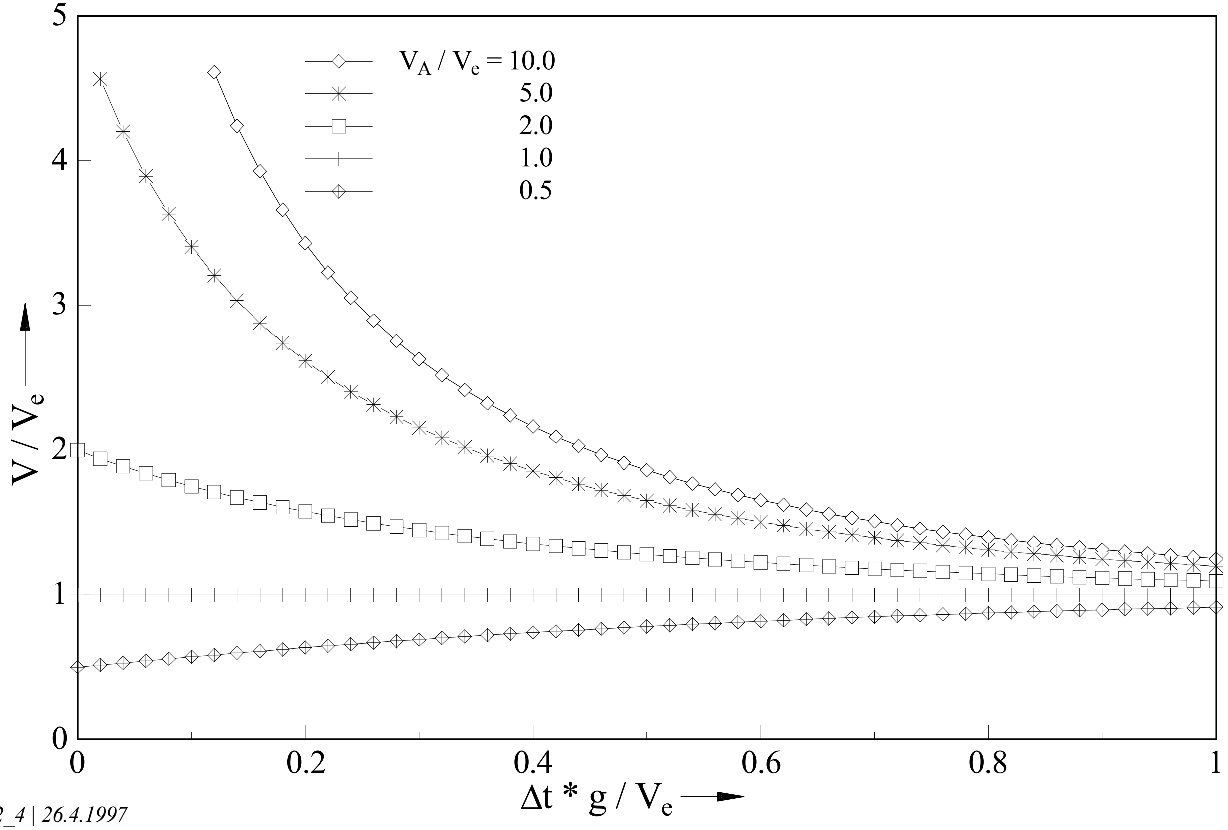


FIG2_4 | 26.4.1997

Altitude change in vertical flight

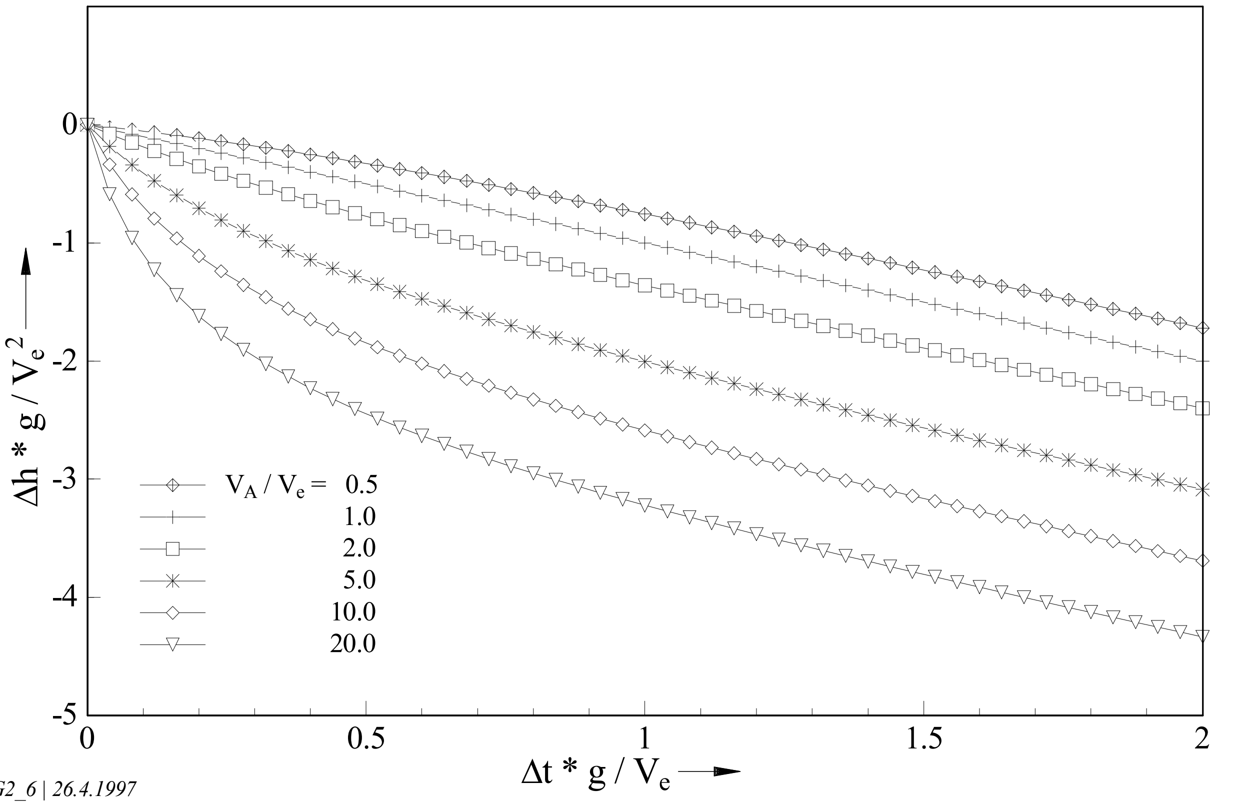


FIG2_6 | 26.4.1997

Task #1: Find suitable parachute

- select parachute: non-oscillating parachute, i.e. **Guide Surface**
- determine parachute diameter:

$\text{Drag} = (C_D S)_e\ \rho/2\ V_e^2 = m\ g$ drag = weight in steady descent

$(C_D S)_e = 40*9.81 / (0.5*1.224*27.3^2) = 0.860\ m^2$ required drag area

$C_D = C_{Dc} = 0.65$ drag coefficient, from parachute handbook

$S_c = 0.86 / 0.65 = 1.324\ m^2$ required parachute constructed area

$D_c = (4 * 1.324 / \pi)^{0.5} = 1.3\ m$ required parachute constructed diameter

$l_s = 1.2 * D_c = 1.65\ m$ selected length of suspension lines

Study Case: Parachute Selection

Task #2: Determine time until steady state

Rough estimate:

It takes about 1 step of non-dimensional time to delerate the system (with non-reefed, fully open parachute).

$\Delta t * g/V_e \approx 1$

$\Delta t \approx V_e / g = 27.3 / 9.81 \approx 2.8$ s

Comparison with 2-DoF simulation
(including free-flight phase without parachute):

$\Delta t \approx 3$ s

Study Case: Deceleration Time

II Parachute Deployment

with constant drag areas $(C_D S)_L$ and $(C_D S)_P$

Snatch Force F_s

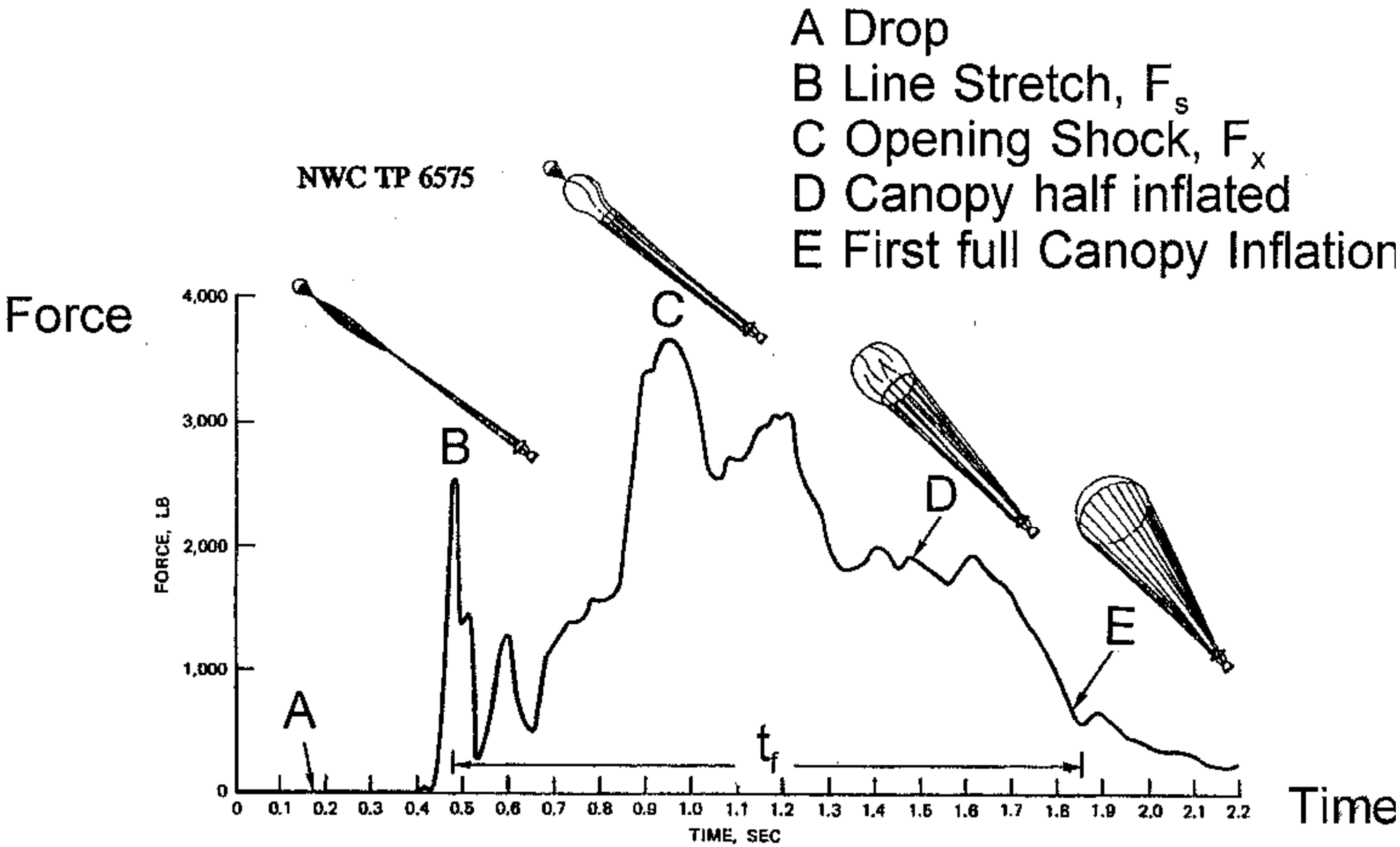


FIGURE 5-46. Opening Process and Opening Force Versus Time for a Guide Surface Personnel Parachute Tested at the El Centro Whirl Tower at 250 Knots With a 200-Pound Torso Dummy.

Parachute Snatch Force

Snatch Force

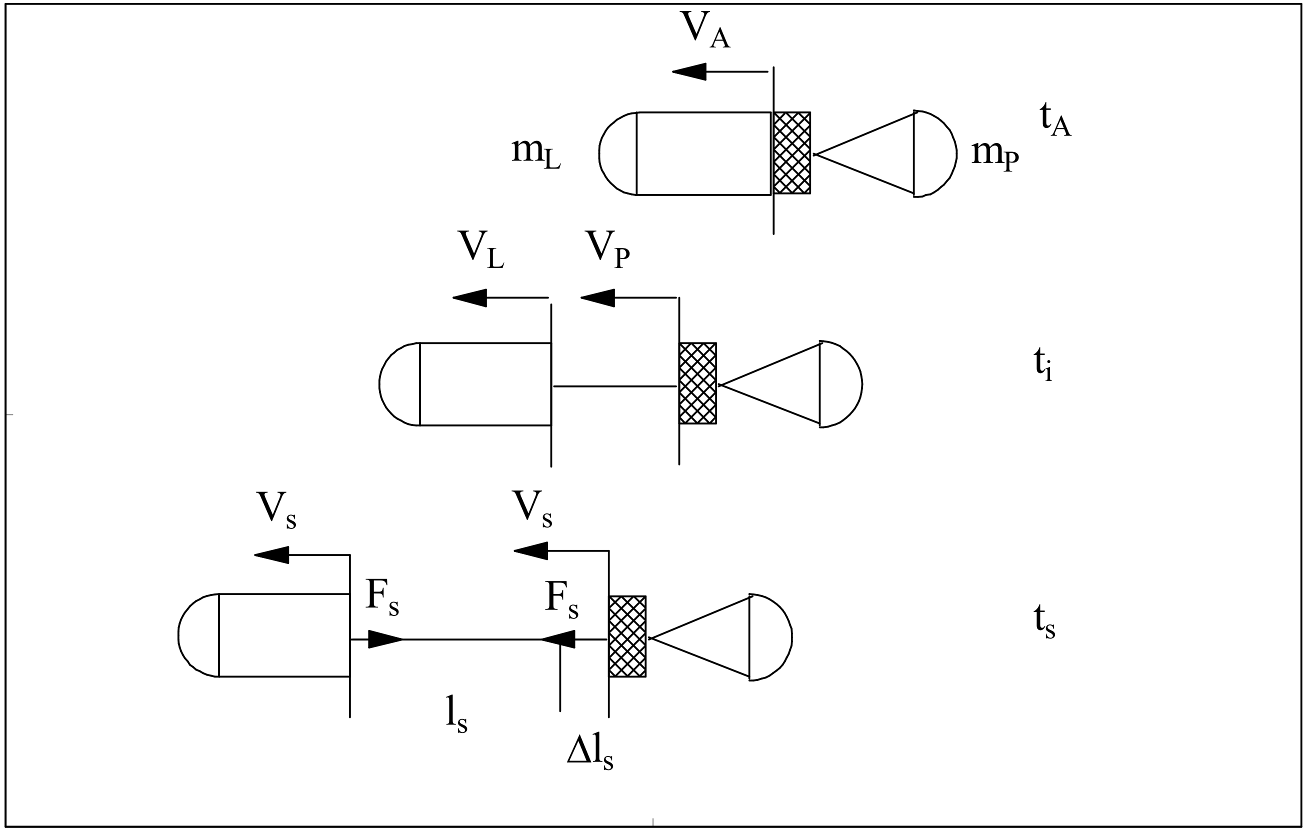


FIG4-1 | 30.4.1997

Snatch Force Estimation (SNATCH)

$$\frac{m_L}{2}V_L^2 + \frac{m_P}{2}V_P^2 = \frac{m_L + m_P}{2}V_s^2 + \Delta E$$

conservation of energy

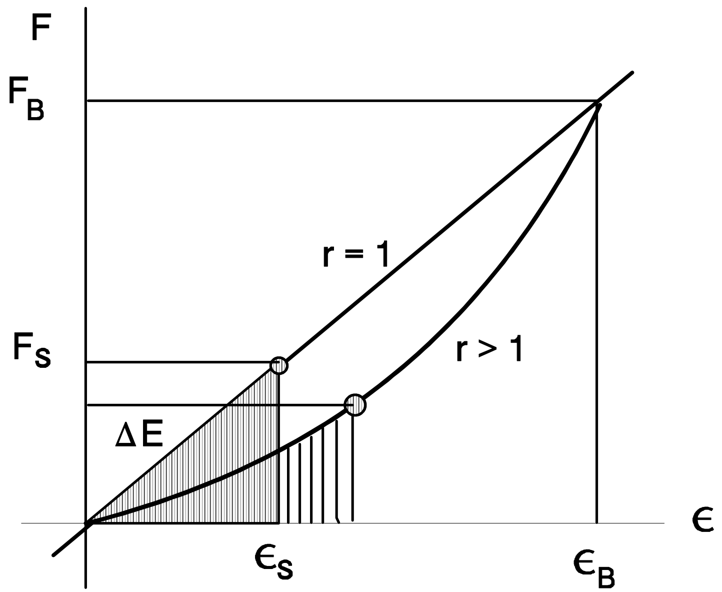
$$m_L V_L + m_P V_P = (m_L + m_P)V_s$$

conservation of momentum

$$\Delta E = \frac{m_L * m_P}{m_L + m_P}\,\frac{\Delta V^2}{2}$$

kinetic energy that gets stored

Energy stored in suspension lines



$$\frac{\Delta V}{V_A} = \frac{1}{1 + t_s / t_L^*} - \frac{1}{1 + t_s / t_P^*}$$ velocity difference at snatch

$$\Delta s = s_L^* \ln(1 + t / t_L^*) - s_P^* \ln(1 + t / t_P^*)$$ distance

$$\Delta s(t = t_s) = \ell_s$$ distance at snatch

where V_A initial velocity at begin of deployment, and

$$s_L^* = \frac{2m_L}{\rho (C_D S)_L} \quad ; \quad s_P^* = \frac{2m_P}{\rho (C_D S)_P}$$

$$t_L^* = \frac{s_L}{V_A} \quad ; \quad t_P^* = \frac{s_P}{V_A}$$

$$\Delta E = n \int_0^{\epsilon_s} F_1 \; \ell_s \; d\epsilon$$

potential energy stored in suspension lines

With

$$F_1 = k \; \ell_s \; \epsilon$$

Hooke's law

$$k = \frac{F_{B1}}{\Delta \ell_B} = \frac{F_{B1}}{\ell_s \; \epsilon_B}$$

spring constant

$$\Delta E = n \; k \; \ell_s^2 \int_0^{\epsilon_s} \epsilon \; d\epsilon = \frac{n}{2} \; k \; \ell_s^2 \; \epsilon_s^2$$

$$F_s = n k \ell_s \epsilon_s = \sqrt{2\ n\ k\ \Delta E}$$

snatch force for linear elongation

For general material properties of the type:

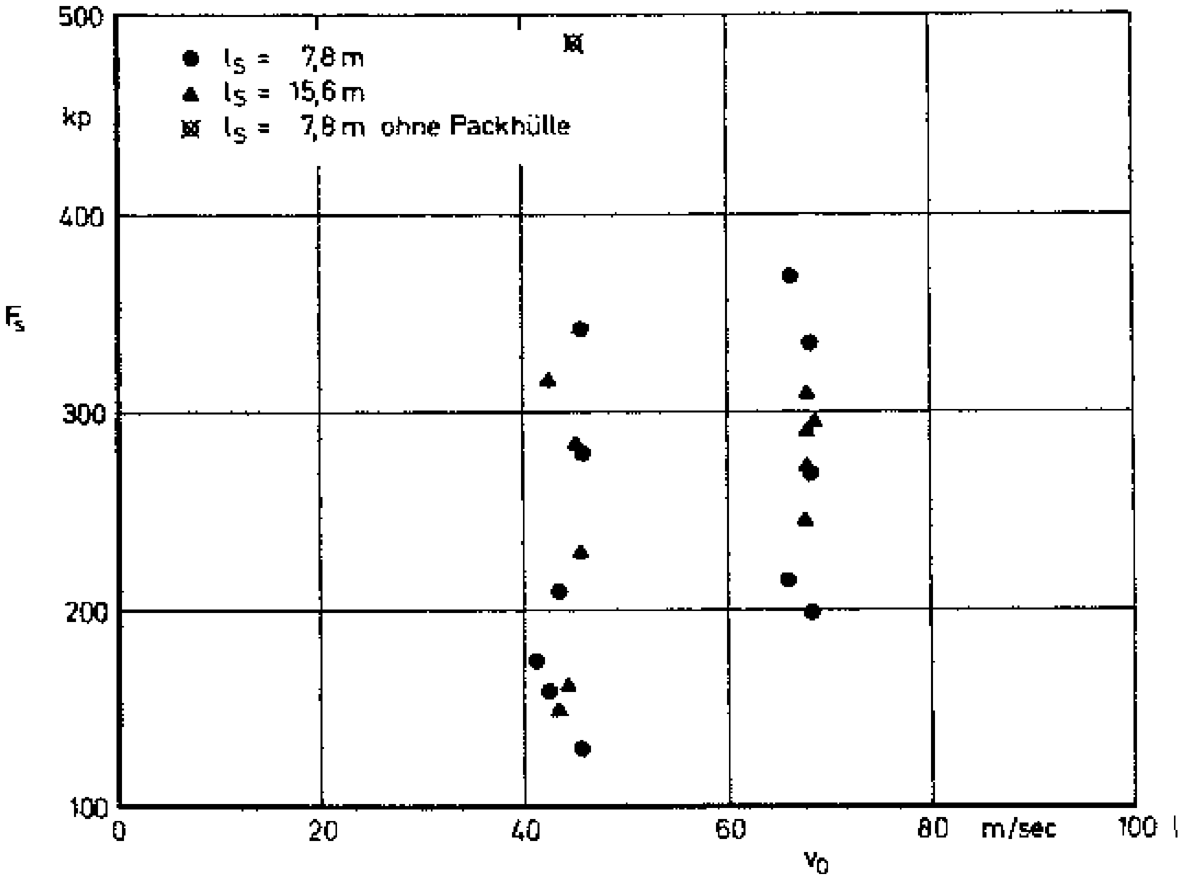
$$F_1 = F_{B1}\ (\ \epsilon\ /\ \epsilon_B\ )^{\ r}$$

r = 1 -> linear material properties (Hooke'sches Law)

r ≠ 1 -> nonlinear material properties

$$F_s = n\ F_{B1} \left[\frac{(r+1)\ \Delta E}{\epsilon_B\ l_s\ n\ F_{B1}} \right]^{\frac{r}{r+1}}$$

Snatch Force



Measured T-10 Snatch Forces (from P. Schuett, DLR-Mitt. 69-11, p. 95)

Task #3 : Estimate Snatch Force (use SNATCH)

1 INPUT File SNATCH.DAT for SNATCH98.EXE
2 for snatch force estimation
3 University of Minnesota Parachute Technology Short Course
4 ______________

1.224	RO	: air density	kg/m^3
39.25	ML	: mass of the load	kg
0.0314	CDSL	: drag area of the load	m^2
0.75	MP	: mass of the parachute pack	kg
0.0314	CDSP	: drag area of the pilot chute	m^2
150.0	V0	: initial speed of both masses	m/s
1.64	LS	: length of suspension line	m
8	N	: number of suspension lines	-
0.30	EPSB	: relative elongation at break	-
6675	FB	: break strength of susp. line	N
1	R	: exponent of strain curve	-
0.0	TA	: initial value of t2	s
0.01	DT	: step size of time	s

Study Case: Snatch Force

Results of SNATCH:

Snatch occurs at t2 = 8.394841E-02 sec dels = 1.639999 m

snatch force	**Fs**	**=**	**10081.66 N**
velocity at snatch	Vs	=	148.412 m/s
difference velocity	delV	=	35.67168 m/s
mass of the load	mL	=	39.25 kg
CDS of the load	CDSL	=	.0314 m*m
mass of the parachute	mP	=	.75 kg
CDS of the parachute	CDSP	=	.0314 m*m
initial velocity	V0	=	150 m/s
suspension line length	ls	=	1.64 m
number of susp. lines	n	=	8 -
relative break length	epsB	=	.3 -
break strength of 1 line	FB	=	6675 N
exponent of strain curve	r	=	1 -

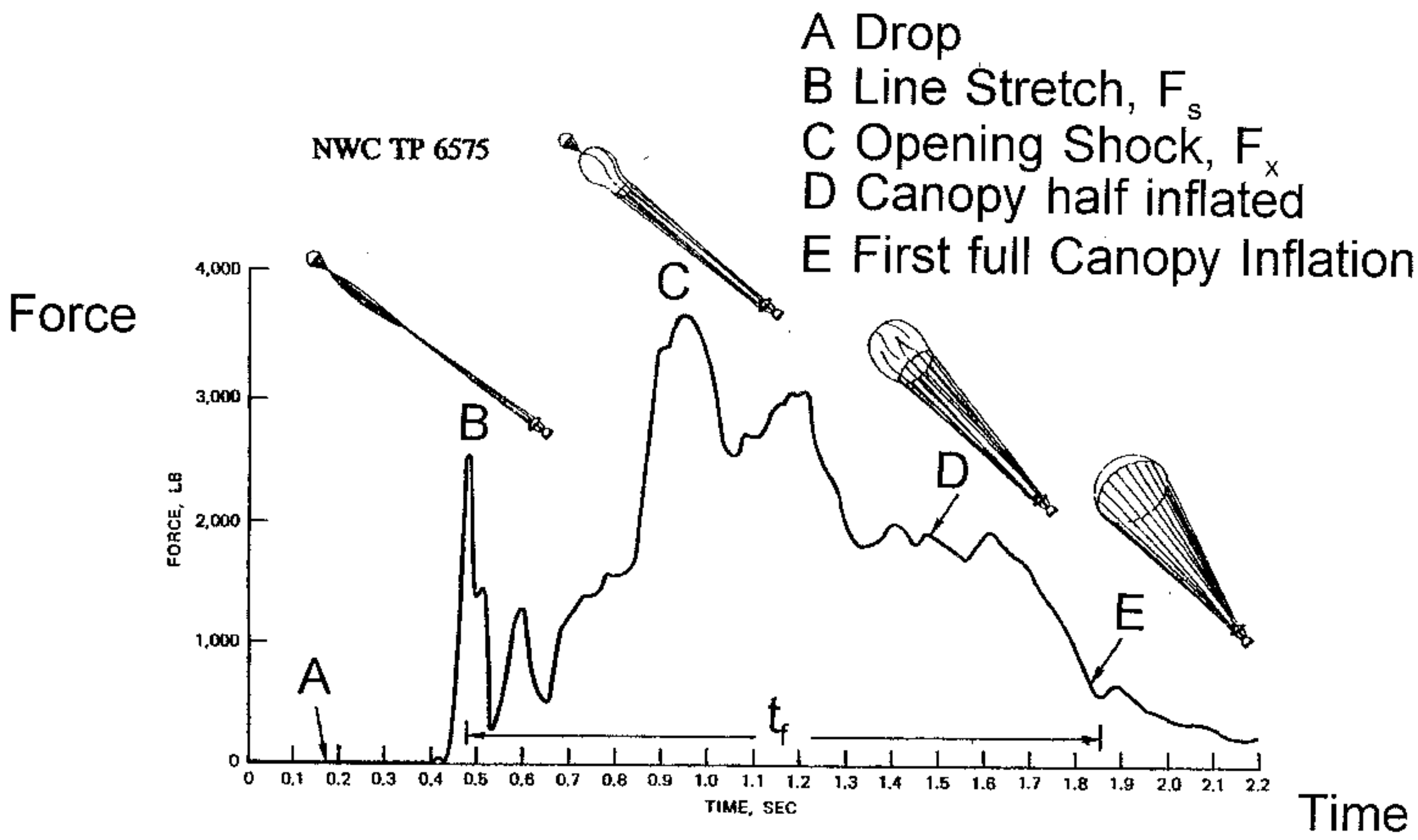
achieved load factor **Fs/n*FB = .1887952** -

Study Case: Snatch Force

III Parachute Inflation

with drasticaly changing drag area $C_DS(t)$

Opening Shock F_X



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Parachute Opening Shock (max. Inflation Force)

Parachute inflation force:

$$F(t) = \frac{\rho}{2} V^2 (C_D S)$$

or, non-dimensional,

$$x(t) = \frac{\frac{\rho}{2} V^2 C_D S}{\frac{\rho}{2} V_s^2 (C_D S)_e} = \left(\frac{V}{V_s}\right)^2 \frac{C_D S}{(C_D S)_e}$$

where V_s = velocity at snatch and $(C_D S)_e$ = steady state drag area.

Parachute Opening Shock (max. Inflation Force)

We are looking for the maximum of x(t), called opening force factor C_K,

$$C_K = x_{\max}(t)$$

Knowing C_K, the opening shock (filling shock) follows from

$$F_x = C_K \frac{\rho}{2} V_s^2 (C_D S)_e$$

Parachute Opening Shock (max. Inflation Force)

Pflanz- Ludtke Method:

Pflanz' (1942):

- introduced analytical functions for the drag area

- integrated the equation of motion in horizontal flight (γ = 0°)

- calculated v(t) and x(t).

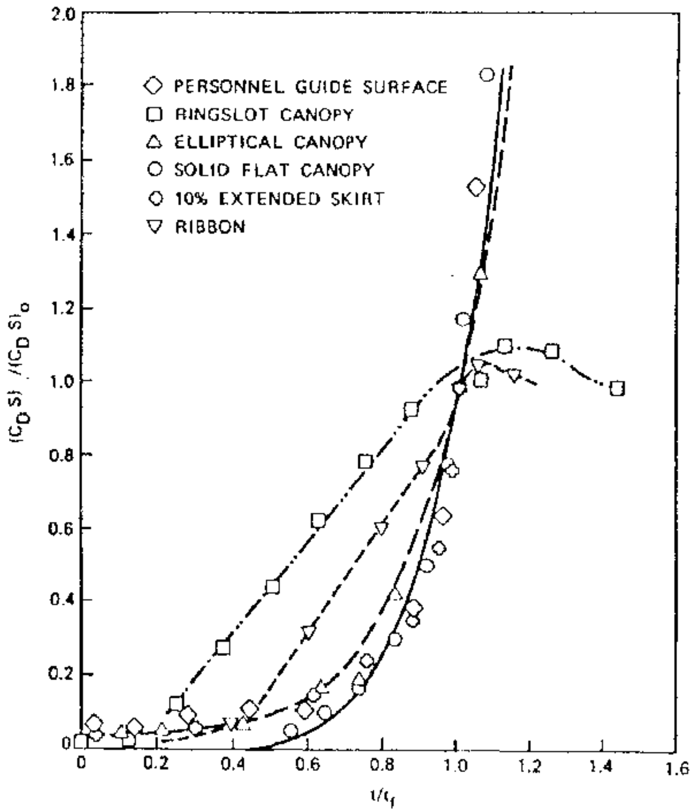
- found closed form expression for C_K

Ludtke (1973): published method in a modified form in English

Doherr (2003): extended method for arbitrary flight path angle γ .

Parachute Opening Shock (max. Inflation Force)

$\frac{C_D S}{(C_D S)_e}$



t / t_f

Normalized Drag Areas vs. Normalized Time

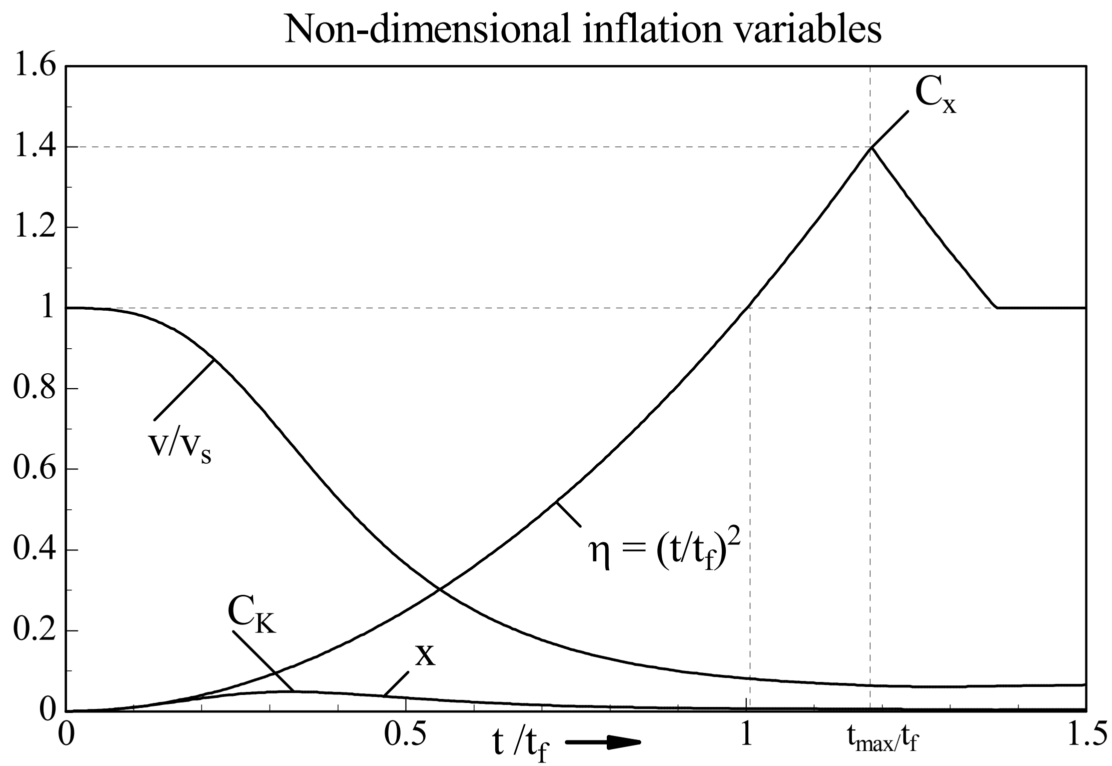
Assume polynomial change of the drag area with time:

$$\eta(t) = \frac{C_D S}{(C_D S)_e} = \left(\frac{t}{t_f}\right)^j$$

$$\frac{t}{t_f} = 0 : \eta = 0; \quad \frac{t}{t_f} = 1 : \eta = 1; \quad \frac{t}{t_f} = C_x^{1/j} : \eta = C_x$$

η	non-dimensional drag area
t_f	inflation time,
	time when the drag area reaches $(C_D S)_e$ the first time
j	exponent
C_x	opening force coefficient (over-inflation factor)

Parachute Opening Shock (max. Inflation Force)



NONPAPER | 6.3.2003

Parachute Opening Shock

Introduce ballistic parameters A and B:

$$A = \frac{F_{re}}{n_f} = \frac{V_e^2}{g D_0 n_f} \quad ; \quad B = \frac{F_{rs}}{n_f} = \frac{V_s^2}{g D_0 n_f} = A\left(\frac{V_s^2}{V_e^2}\right)$$

D_0 parachute nominal diameter D_0,
F_{rs} and F_{re} Froude numbers at snatch and at steady state,
n_f non-dimensional inflation time

$$n_f = t_f \frac{V_s}{D_0}$$

V_e steady state velocity of descent v_e, defined by

$$V_e^2 = \frac{2 m_t g}{\rho (C_D S)_e}$$

Parachute Opening Shock (max. Inflation Force)

In horizontal flight (γ = 0°) and for sufficiently large V_s/V_e,
If:

$$A \leq \frac{j+2}{j(j+1)} C_x^{(j+1)/j}$$

then

$$C_{K0} = [\frac{j+2}{2(j+1)}]^2 \; [\frac{j(j+1)A}{j+2}]^{j/(j+1)}$$

else

$$C_{K0} = [1 + \frac{1}{A(j+1)} C_x^{(j+1)/j}]^{-2} \; C_x$$

Parachute Opening Shock (max. Inflation Force)

If $\gamma \neq 0$, or if V_s/V_e is small, then

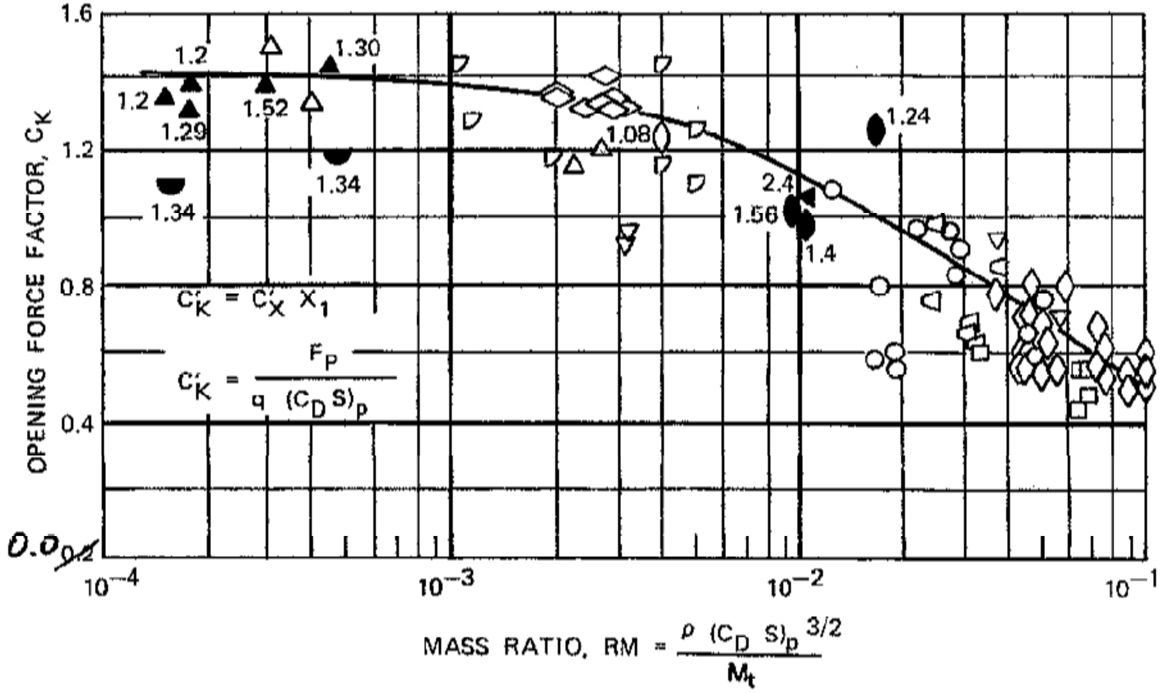
$$C_K = C_{K0} + C_1 + C_2$$

where

$$C_1 = \sqrt{j}\,\left(\frac{V_e}{V_s}\right)^2 e^{-B}$$

$$C_2 = \sqrt{j}\,\left(\frac{V_e}{V_s}\right)^2 (1 - e^{-B}) \sin(-\gamma_0)\, e^{-\frac{A}{6} j^{0.25}}$$

Parachute Opening Shock (max. Inflation Force)



Opening Force Factor C_K vs. Mass Ratio of reefed Parachutes

(from Knacke, NWC TP 6575, p5-57)

Task # 4: Estimate opening shock (use Fx05)

1 INPUT File Fx05.DAT of Fx05.EXE
2 for opening shock estimation
3 Pflanz method, see AIAA paper 2003-2173
4 ______________

1.224	RO	: air density	kg/m^3
9.81	g	: gravity constant	m/s^2
1.373	D0	: parachute nominal diameter	m
40.0	M	: system mass	kg
0.86	CDSe	: steady state system drag area	m^2
10.9	nf	: non-dimensional inflation time	-
6	j	: polynomial exponent	-
1.7	Cx	: opening force coefficient	-
149.3	Vs	: snatch velocity	m/s
10	gammas	: initial flight path angle	deg

Study Case: Opening Shock

Results:

Opening shock	**Fx**	**=**	**17973.39**	**N**
Load factor	nx	=	45.80374	-
ballistic parameter	A	=	5.078238	-
ballistic parameter	B	=	151.8693	-
Opening force factor	CK	=	1.531595	-
Uncorr opening force factor	C0	=	1.535377	-
Corr. factor for small Vs/Ve	C1	=	0	-
Corr. factor for gammas <> 0	C2	=	-3.781822E-03	-
air density	RO	=	1.224	kg/m^3
gravity constant	g	=	9.81	m/s^2
parachute nominal diameter	D0	=	1.373	m
system mass	m	=	40	kg
steady state system drag area	CDSe	=	.86	m^2
steady state velocity	Ve	=	27.30484	m/s
non-dimensnl inflation time	nf	=	10.9	-
polynomial coefficient	j	=	6	-
opening force coefficient	Cx	=	1.7	-
velocity at snatch	Vs	=	149.32	m/s
flight path angle at snatch	gammas	=	10	deg

Study Case: Opening Shock

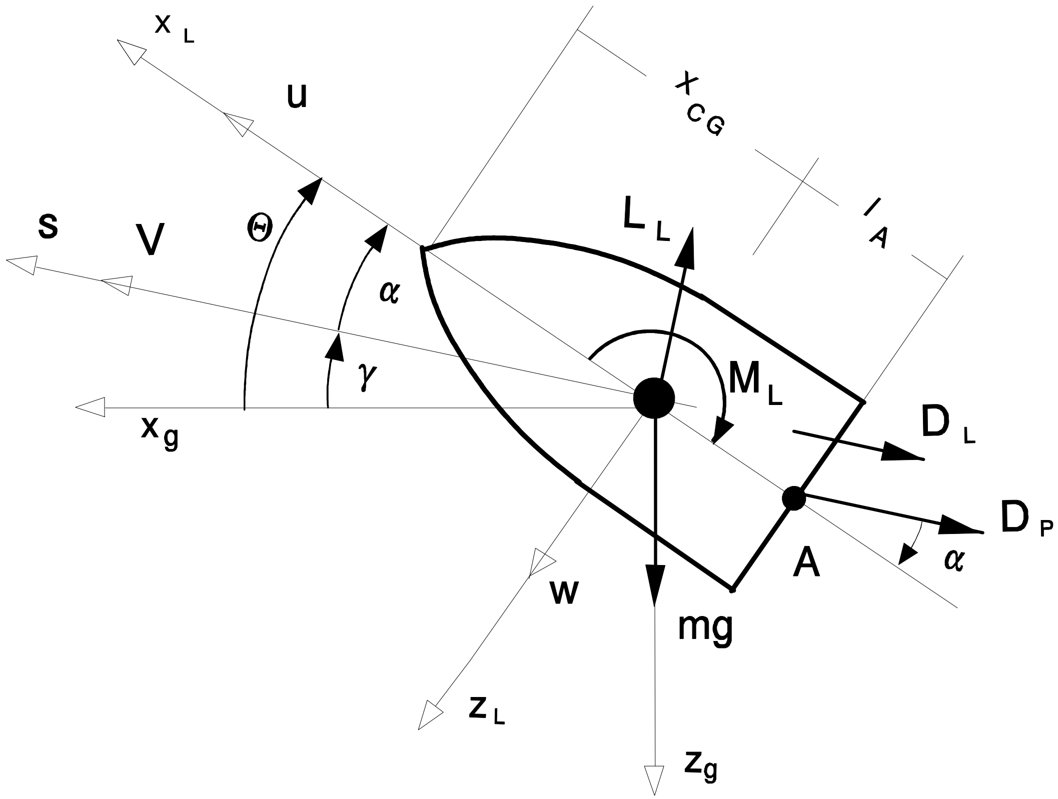
IV System Deceleration and Stabilisation

with constant drag area C_DS

Velocity decreasing

Oscillation building up

Stability analysis of 3DoF-System (OSCILALx) with variables s, γ, and Θ
Introduction of the parachute by the drag force D_P only



3DoF-equations of motion in trajectory coordinates:

$$m \frac{dV}{dt} = - mg \sin\gamma - D_L - D_P$$

$$mV \frac{d\gamma}{dt} = - mg \cos\gamma + L$$

$$I_y \frac{d^2\Theta}{dt^2} = M_L - \ell_A D_P \sin\alpha$$

$$\frac{ds}{dt} = V \; ; \qquad \frac{d\Theta}{dt} = q$$

$$t = t_A \; : \; s = s_A \; ; \; V = V_A \; ; \; \gamma = \gamma_A \; ; \; \Theta = \Theta_A \; ; \; q = q_A$$

Stability analysis

Case: Horizontal flight:
γ_A = 0; ρ = const; gravity negelected: g = 0

The linearized equations of motions have two eigenmodes:

1. Exponential decay of the velocity along the flight path s

$$V = V_A \, e^{-\frac{\varrho \, C_D S}{2m}(s - s_A)}$$

2. Angle of Attack Oscillation

$$\alpha(s) = \alpha_A \, e^{\lambda s} = \alpha_A \, e^{\delta s} \sin(\omega s)$$

Damping and frequency:

$$\delta = \frac{1}{2}\,\frac{\varrho\, S_L}{2m}\left[\, C_{DL} + \frac{(C_D S)_P}{S_L} - C_{L\alpha} + \frac{m\, d_L^{\,2}}{2\, I_y}\, C_{mLq} \right]$$

$$\omega^2 = \frac{\varrho\, S_L\, d_L}{2\, I_y}\left[\, -\, C_{mL\alpha} + \frac{(C_D S)_P}{S_L}\,\frac{\ell_A}{d_L} \right]$$

Conditions for dynamic stability:

	<	0	amplitude decreases (stabil)
δ	=	0	amplitude remains constant (neutral stability)
	>	0	amplitude increases (instable)

Parachute **reduces damping** because of + $(C_D S)_P / S_L$ - term

Parachute **increases oscillation frequency**

Task #5a: Estimate Dynamic Stability in Horizontal Flight (Use OSCILALH.BAS)

```
170  DATA 0.351  : READ TA: REM initial time
180  DATA 122.5  : READ VA: REM initial velocity
190  DATA 0.200  : READ DL: REM reference diameter of the load
200  DATA 40.00  : READ M: REM mass of the load
210  DATA 2.1     : READ IY: REM moment of inertia about y-axis
220  DATA 0.400  : READ XH: REM attachment point of parachute
230  DATA -1.00  : READ CXL0: REM CX of load at alfa = 0
240  DATA -2.78  : READ CZLALF: REM dCZL/dalfa of load
250  DATA 1.11   : READ CMLALF: REM dCML/dalfa of load
260  DATA -10.0  : READ CMLQ: REM pitch damping deriv. of load; q*D/2V
270  DATA  0.0   : READ CMPQ: REM pitch damping derivative of parachute
280  DATA 0.65   : READ CDP0: REM drag coeff. of chute at alfa = 0
290  DATA 0.96   : READ CDSP: REM drag area of chute
```

Results of OSCILALH.BAS in horizontal flight

Initial velocity	VA	=	122.5	m/s
load diameter	DL	=	.2	m
system mass	M	=	40	kg
moment of inertia	Iy	=	2.1	kg*m*m
chute attachment point	XH	=	.4	m
load aerodynamics:	CXL0	=	-1	-
	CZLALF	=	-2.78	-
	CMLALF	=	1.11	-
	CMLQ	=	-10	-
parachute data:				
drag coefficient	CDP0	=	.65	-
drag area	CDSP	=	.96	-
pitch damping coeff.	CMPQ	=	0	-
diameter	DP	=	1.371305	m

angle of attack oscillation:

CDSP	DELTA [1/m]	OMEGA*i [1/m]
.96	6.24099E-03	.3314168
VA [m/s]	Ve [m/s]	l[m]
122.5	25.43085	18.95856

Study Case: Dynamic Stability

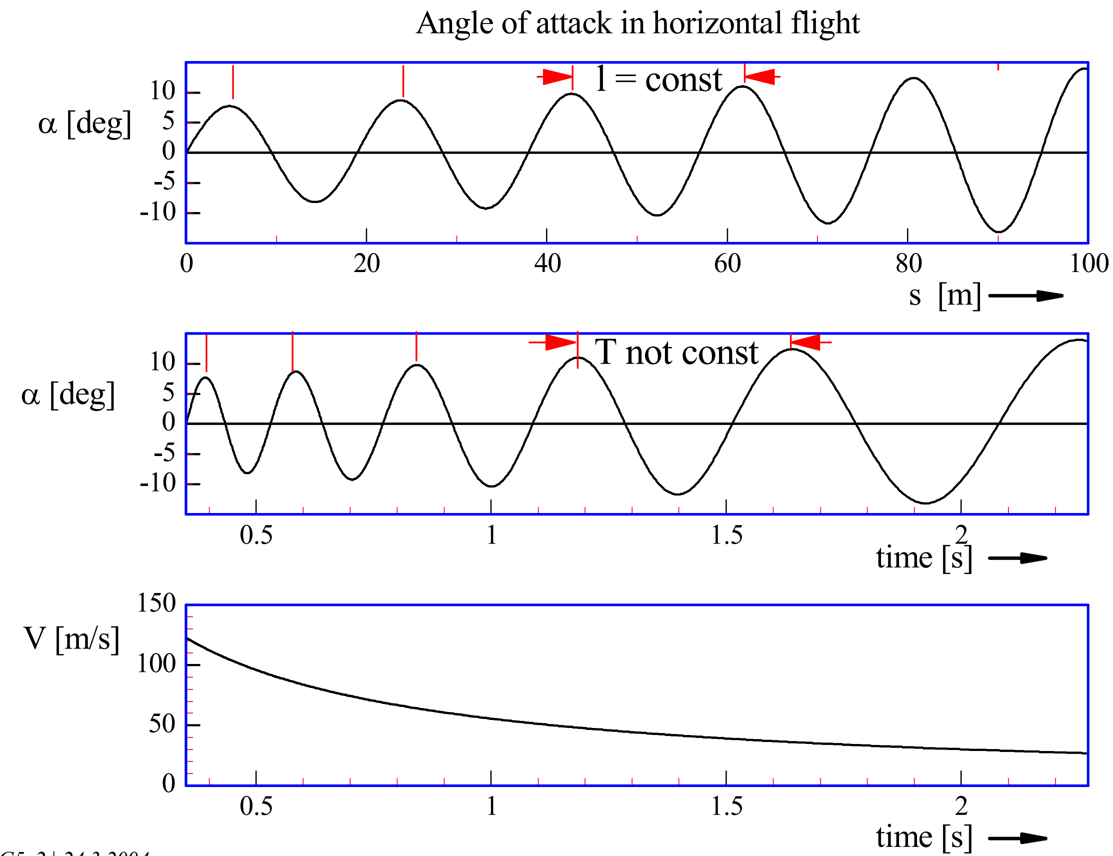


FIG5_2 | 24.3.2004

Study Case: Dynamic Stability

Stability analysis

Case: Vertical flight:

γ_A = -90°; $V(-90°) = V_e$; ρ = const; g = 9.81 m/s^2

After small disturbances ΔV_A, $\Delta\Theta_A$, and $\Delta\gamma_A$ from vertical flight:

1. Exponential decay of the velocity

$$\Delta V = \Delta V_A \, e^{-2(t - t_A) g / V_e}$$

2. Vertical glide motion

$$\Delta\gamma = \Delta\gamma_A \, e^{(\delta \pm i\omega)s} ; \quad \Delta\Theta = \Delta\Theta_A \, e^{(\delta \pm i\omega)s}$$

$$\omega = 0; \quad \delta = -g/V_e^2$$

3. Pendulum motion

$$\delta \approx - \frac{\varrho \ S_L}{4m} \left[\ C_{L\alpha} \ - \ \frac{m \ d_L^{\,2}}{2 \ I_y} \ C_{mLq} \ \right]$$

$$\omega^2 = - \frac{\varrho \ S_L \ d_L}{2 \ I_y} \left[\ C_{mL\alpha} \ - \ \frac{(C_D S)_P}{S_L} \ \frac{\ell_A}{d_L} \ \right]$$

The damping of the vertical pendulum motion is provided by the aerodynamic damping of the payload!

Task # 5b: Estimate Dynamic Stability in vertical flight (Use OSCILALV.BAS)

```
180  DATA 24.4    : READ VA: REM initial velocity
190  DATA 0.200   : READ DL: REM reference diameter of the load
200  DATA 40.00   : READ M: REM mass of the load
210  DATA 2.1     : READ IY: REM moment of inertia about y-axis
220  DATA 0.400   : READ LA: REM attachment point of parachute
230  DATA -1.00   : READ CXL0: REM CX of load at alfa = 0
240  DATA -2.78   : READ CZLALF: REM dCZL/dalfa of load
250  DATA 1.11    : READ CMLALF: REM dCML/dalfa of load
260  DATA -10.0   : READ CMLQ: REM pitch damping deriv. of load; q*D/2V
270  DATA  0.0    : READ CMPQ: REM pitch damping derivative of parachute
280  DATA 0.65    : READ CDP0: REM drag coeff. of chute at alfa = 0
290  DATA 0.96    : READ CDSP: REM drag area of chute
```

Study Case: Dynamic Stability

Results of OSCILALV.BAS in vertical flight

Initial velocity	VA	=	122.5	m/s
load diameter	DL	=	.2	m
system mass	M	=	40	kg
moment of inertia	Iy	=	2.1	kg*m*m
chute attachment point	XH	=	.4	m
load aerodynamics:	CXL0	=	-1	-
	CZLALF	=	-2.78	-
	CMLALF	=	1.11	-
	CMLQ	=	-10	-
parachute data:				
drag coefficient	CDP0	=	.65	-
drag area	CDSP	=	.96	-
pitch damping coeff.	CMPQ	=	0	-
diameter	DP	=	1.371305	m

angle of attack oscillation:

CDSP	DELTA [1/m]	OMEGA*i [1/m]	
.96	-.1343E-02	0.3315E+00	in trajectory coordinates
	DELTAt [1/s]	OMEGAt*i [1/s]	
	-.3416E-01	0.8430E+01	in the time domain

VA [m/s]	Ve [m/s]	l [m]	T[s]
24.4	25.43085	18.95535	.7453685

Study Case: Dynamic Stability

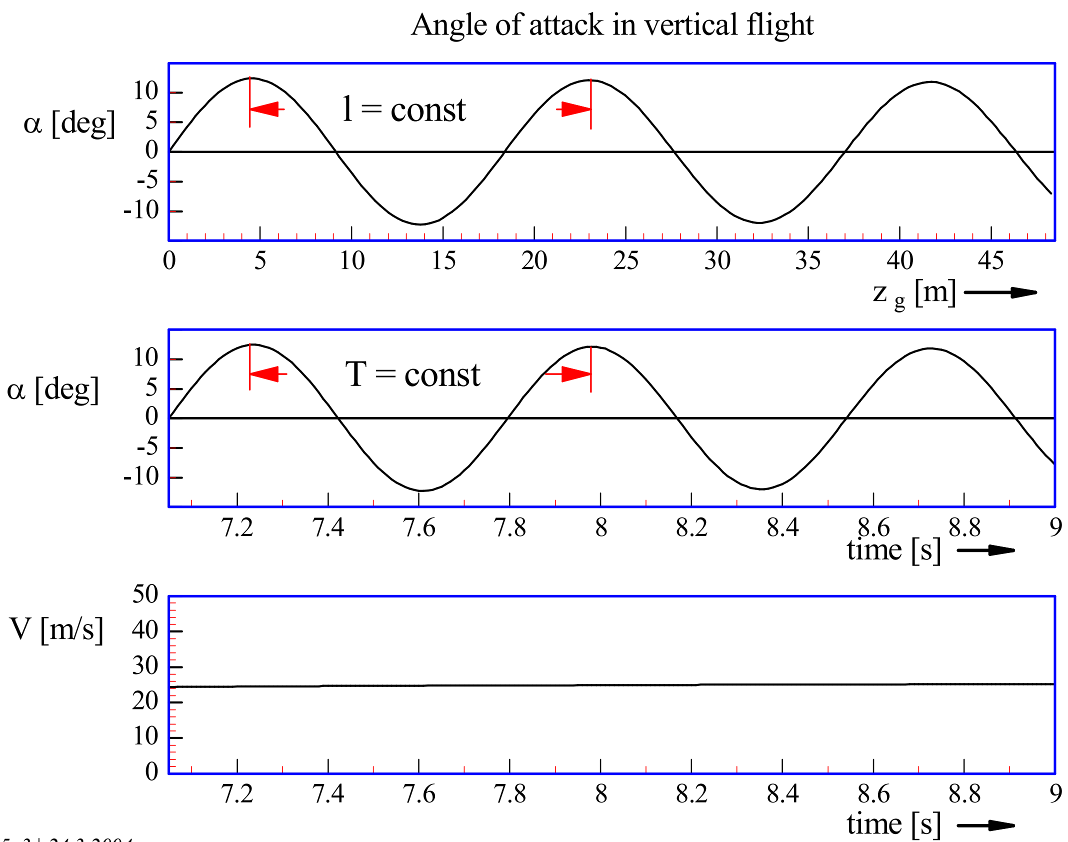


FIG5_3 | 24.3.2004

Study Case: Dynamic Stability

Task #6: Trajectory simulation (use 2DOFT05)

Numerical integration of non-linear differential equations using Pflanz' drag area model:

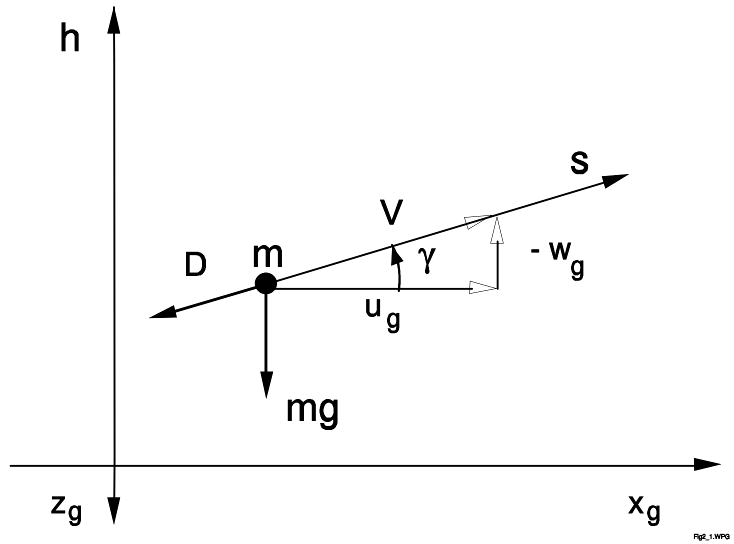
$$m\ \dot{u}_g = -\frac{\rho}{2} V^2 C_D S \cos\gamma$$

$$m\ \dot{w}_g = \frac{\rho}{2} V^2 C_D S \sin\gamma + m\ g$$

$$\dot{x}_g = u_g$$

$$\dot{z}_g = w_g$$

$$t = t_A: \quad x_g = x_A; \quad z_g = z_A; \quad u_g = u_A; \quad w_g = w_A$$



Study Case: Trajectory

Drag area model:

$$C_D S(t) = [\ \eta_a + (1 - \eta_a)(\ t\ /\ t_{fmax}\ )^{j_f}\ ] * (C_D S)_e$$

$$\eta_a = (C_D S)_a\ /\ (C_D S)$$

$(C_DS)_a$ drag area at the beginning of the filling or disreefing

$(C_DS)_e$ drag area at the end of the filling phase

t_{fmax} filling time

j_f exponent of the filling characteristic

Study Case: Trajectory

Task #6: Input data

```
2DOFT05.DAT for Pascal program 2DOFT05.PAS (of June 2005)
Study Case
Parachute work shop 2005
deceleration of 40 kg system by 4.5 ft Guide Surface Parachute
18605         ID-number
  initial data:
9.81      g0        gravity constant
0.0       TAU0      inital time
0.001     DT        step size for integration
10.0      TAUE      final time
10        noutput   every noutput'th data point is stored
0.0       XG0       range
100.0     HG0       altitude
147.7     VXG0      horizontal velocity
26.0      VHG0      vertical velocity, upwards positive
```

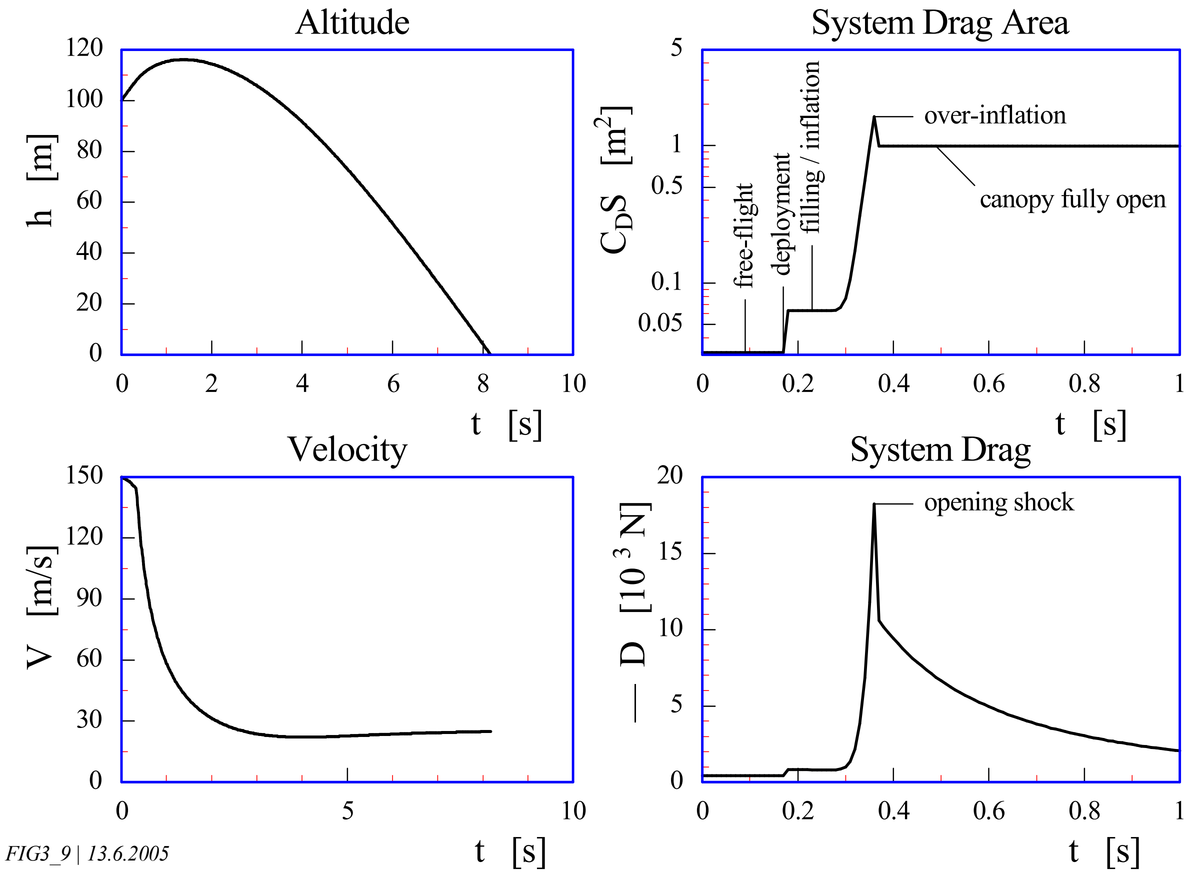
system data for stage: System free flight

1	STAGE	stage number
DT	controlls end of stage (perm inputs: characters T, DT, H, DH)	
0.17	value of (T , DT, H , DH) that initiates next stage	
40.0	MASS	mass (kg)
0.0314	CdS_payload (m*m)	cylindrical payload
0.0	S0	total refer. area (m*m) = S0max of all parachutes
0.0	CD	para drag coeff = CDr (reefed) or CD0 (disreefed)
0	nc	number of parachutes in cluster (=1,2,3..)
n	FILLING	= y or n (if y then variable drag area)
tf	permitted inputs: characters tf (=tfmax) or nf	
0.0	value of (tf or nf); if nf then tfmax=nf*sqrt(4*S0/nc/pi)/V	
0.0	ETA = (CdS)a / (CdS)e = initial/final drag area	
0.0	JF	filling exponent in (t/tfmax)^jf
0.0	Cx	opening force coefficient

system data for stage: deployment of 4.5 ft Guide Surface parachute

2	STAGE	stage number
DT	controlls end of stage (permitted inputs: T , DT, H , DH)	
0.08	value of (T , DT, H , DH) that initiates next stage	
40.0	MASS	mass (kg)
0.0314	CdS_payload (m*m)	
0.0314	S0	total refer. area (m*m) = S0max of all parachutes
1.0	CD	para drag coeff = CDr (reefed) or CD0 (disreefed)
1	nc	number of parachutes in cluster (=1,2,3..)
n	FILLING = y or n (if yes then variable drag area)	
nf	permitted inputs: characters tf (=tfmax) or nf	
0.0	value of (tf or nf); if nf then tfmax=nf*sqrt(4*S0/nc/pi)/V	
0.0	ETAA	= (CdS)a / (CdS)e = initial/final drag area
0.0	JF	filling exponent in (t/tfmax)^jf
0.0	Cx	opening force coefficient

```
  system data for stage: parachute inflation+system deceleration+steady descent
3          STAGE  stage number
T          controlls end of stage (permitted inputs: T , DT, H , DH)
10.0       value of (T , DT, H , DH) that initiates next stage
40.0       MA       mass (kg)
0.0314     CdS_payload (m*m)
1.48       S0       total refer. area (m*m) = S0max of all parachutes
0.65       CD       para drag coeff = CDr (reefed) or CD0 (disreefed)
1          nc       number of parachutes in cluster (=1,2,3..)
y          FILLING = y or n (if yes then variable drag area)
tf         permitted inputs: characters tf (=tfmax) or nf
0.1        value of (tf or nf); if nf then tfmax=nf*sqrt(4*S0/nc/pi)/V
0.0327     ETA = (CdS)a / (CdS)e = initial/final drag area
6.0        JF       filling exponent in (t/tfmax)^jf
1.7        Cx       opening force coefficient
  system data for stage:
0             STAGE      stage number (if STAGE = 0, then simulation terminates)
```



Study Case: Trajectory